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Factoring Decision Tree

Factor out the Common factor

Example
$2x^2y - 4xy^2 + xy$
$= xy(2x - 4y + 1)$

Example
$4x^{\frac{1}{2}}(x-2)^{\frac{-3}{4}} + 2x^{\frac{-1}{2}}(x-2)^{\frac{1}{4}}$
$= 2x^{\frac{-1}{2}}(x-2)^{\frac{-3}{4}}\left(2x + (x-2)\right)$
$= 2x^{\frac{-1}{2}}(x-2)^{\frac{-3}{4}}(3x-2)$

Identify the number of terms

Two Terms

Difference of Squares
$A^2 - B^2 = (A-B)(A+B)$

Example
$4x^2 - 81y^2 = (2x)^2 - (9y)^2$
$= (2x - 9y)(2x + 9y)$

OR

Difference of Cubes
$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$

Example
$8x^3 - 1 = (2x)^3 - (1)^3$
$= (2x-1)\left((2x)^2 + (2x)(1) + (1)^2\right)$
$= (2x-1)(4x^2 + 2x + 1)$

OR

Sum of Cubes
$A^3 + B^3 = (A+B)(A^2 - AB + B^2)$

Example
$125x^3 + 27z^3 = (5x)^3 + (3z)^3$
$= (5x + 3z)\left((5x)^2 - (5x)(3z) + (3z)^2\right)$
$= (5x + 3z)(25x^2 - 15xz + 9z^2)$

Note: a sum of squares $A^2 + B^2$ does not factor in the real numbers

Three Terms

Other formulas
$A^2 + 2AB + B^2 = (A+B)^2$
$A^2 - 2AB + B^2 = (A-B)^2$

Factoring trinomials with a leading 1
Look for two numbers that:
multiply to c
and add to b
Call them p and q
Re-write the polynomial as the product

$x^2 + bx + c$
$= (x+p)(x+q)$

Example
$x^2 - 4x - 12$
$= (x-6)(x+2)$

$* = -12$ Options
$+ = -4$ 1, -12
$p = -6$, 2, -6
$q = 2$ 3, -4

Factoring trinomials with a non-leading 1
Look for two numbers that:
multiply to the product ac
and add to b
Call them p and q
Re-write the polynomial
Factor by grouping the first two terms and the last two terms
Then factor the common factor that is left

$a \neq 0 \quad ax^2 + bx + c$
$= ax^2 + px + qx + c$

Example
$4x^2 + 7x - 15$
$= 4x^2 + 12x - 5x - 15$
$= 4x(x+3) - 5(x+3)$
$= (4x-5)(x+3)$

$* = (4)(-15)$
$= -60$
$+ = 7$
Options
-1*60 -2*30
-3*20 -4*15
-5*12 -6*10

$p = -5$,
$q = 12$

Four Terms

Factor by grouping
Option 1
Group the first two terms and the second two terms
$x^3 - 5x^2 + 3x - 15$
$= x^2(x-5) + 3(x-5)$
$= (x-5)(x^2+3)$

Option 2
Group three terms
$x^2 - 4x + 4 - y^2$
$= (x-2)^2 - y^2$ (a difference of squares)
$= [(x-2) - y][(x-2) + y]$
$= (x - 2 - y)(x - 2 + y)$

Always check to see if you can factor more!