Logical Operators

This sheet is just a broad overview of some of the terminology of logic. For a more in depth discussion please look through additional resources. *Discrete and Combinatorial Mathematics* by R. Grimaldi contains an accessible chapter on logic which will provide the reader with a much more thorough understanding of the basics of logic. Another book worth looking at is *Set Theory and Logic* by R. Stoll. These books and many other on the subject, ranging from introductory to advanced, are available in the Geoffrey R. Weller Library at UNBC.

1 Propositions

Logical connectives are operators that operate on *propositions*. A proposition is a statement (declarative sentence) that is either true or false *but not both*. Examples of propositions are "The sky is blue", "1=6", and even "The LSC is on the 2nd floor of the Teaching and Learning Building". From now on we will be using letters to stand for propositions. For example: p: the sky is blue q: the grass is green Now in order to write "The sky is blue *and* the grass is green" we can simply write "p *and* q". This will make expressing our ideas a lot easier.

2 Operators

Now that we have these things we call propositions we need ways of putting them togethor. Really what we are talking about is building compound statements. This is much like how we start out with numbers and letters representing unknowns and then use the operations of arithmetic to put togethor ordinary algebraic expressions.

2.1 Not

The simplest thing we can do to a proposition is negate it. The process of doing this is called *negation*. If the original statment was true then the negated statement is false and vice versa. We notate *not* p by writing $\neg p$ though some writers denote this as p', or sometimes $\bar{p}$.

Now we will introduce a way of defining operators using a truth table. In the leftmost column we see the possible truth values of p (True-T, False-F) and in the right column we see the corresponding value of $\neg p$.

p	$\neg p$
T	F
F	T

2.2 And

Another familiar operator is AND, or more formally *conjuction*. p *and* q is usually denoted $p \wedge q$. An alternative notation is $p \cdot q$ or even pq. The values for AND are shown in the truth table below

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2.3 Or

The final primative operator is OR which is formally called *disjuction.* p *or* q is usually denoted $p \vee q$ or sometimes $p + q$. The behaviour of OR is shown below:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

An important thing to notice is that if p is true and q is true then p *or* q is true. In conversational English we often use or in the exclusive sense, that is p *or* q is taken to mean p *or* q *but not both*, but in mathematics we use an inclusive or unless otherwise specified.

2.4 Implies

The last connective we will talk about is IMPLIES. *Implication* occurs often so we give extra attention to it. p *implies* q is denoted $p \to q$. Here is the truth table for implication:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

When we say p *implies* q we are really saying the same thing as q *or not* p as shown in the next table. Notice how the two rightmost columns match.

p	q	$\neg p$	$\neg p \vee q$	$p \to q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

There are many different ways that we say p *implies* q. Just think of them as "logical synonyms". The following all mean the exact same thing as p *implies* q.

1. *If* p*, then* q
2. p *only if* q
3. p *is sufficient for* q
4. q *is necessary for* p

It is very important to understand that $p \to q$ is not the same thing as $q \to p$. Perhaps the easiest way to make sense of this is through a sentence. Prince George's strange weather aside, we know that *If it is raining, then it is cloudy*; however, we can agree that the statement *If it is cloudy,then it is raining* is not true, the sky could simply be overcast.

You will probably hear of things like *contrapositive*, *converse*, and *inverse* when talking about implication. These are really just different rearrangements of some original implication. The meaning is shown below.

Original	Contrapositive	Converse	Inverse
$p \to q$	$\neg q \to \neg p$	$q \to p$	$\neg p \to \neg q$

You might also hear of p *if and only if* q, denoted $p \leftrightarrow q$ and sometimes written p *iff* q. This is just the same thing as $(p \to q) \wedge (q \to p)$ Essentially when we state $p \leftrightarrow q$ we are saying "p is logically the same thing as q". Of course this in itself is a proposition which means that it may or may not be true.