Using Polarity Tables Worksheet

When to use it:

- Whenever you are solving an inequality that is not linear this includes when finding domains of even indexed roots
- Finding intervals of increasing and decreasing and concavity

Steps to follow:

<table>
<tr><td>Process:</td><td>Example:</td></tr>
<tr><td>

- What are you trying to solve?

</td><td>Where is $\frac{2x^3-5x^2-12x}{x-6} \geq 0$</td></tr>
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- Make the inequality an equation and solve it and determine where it does not exist
- If it is a rational expression set the denominator equal to zero as well and use any solutions there in your intervals as well
- Use those numbers to make intervals (a number line may help) and set up the table fill in the appropriate sign for the intervals and then the full equation.

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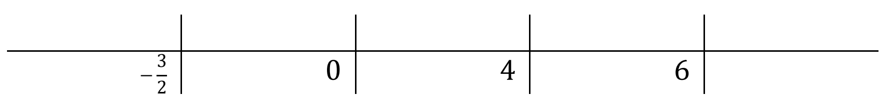
$\frac{2x^3-5x^2-12x}{x-6} = 0$
Consider
$\frac{2x^3-5x^2-12x}{x-6}$
The function equals zero when:
$2x^3-5x^2-12x=0$
$x(2x^2-5x-12)=0$
$x(2x^2-8x+3x-12)=0$
$x[2x(x-4)+3(x-4)]=0$
$x(x-4)(2x+3)=0$
$x=0 \text{ or } x-4=0 \text{ or } 2x+3=0$
$x=0 \text{ or } x=4 \text{ or } x=-\frac{3}{2}$
The function DNE when:
$x-6=0$
$x=6$
So use $x=4$ $x=-\frac{3}{2}$ $x=0$ $and\ x=6$ to create intervals



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- Fill in the intervals (top row) and the factored pieces (first column)
- The last entry in the first column is the original factored statement
- The test number can be any number in the interval
- Use the test number to determine the signs of the individual entries
- This is done by substituting the x value (test number) in the expression with the test number. For example: use $x=-2$ in $x-4$ to get $(-2)-4$ which gives a

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<table>
<tr><td>Interval</td><td>$\left(-\infty,-\frac{3}{2}\right)$</td><td>$\left(-\frac{3}{2},0\right)$</td><td>$(0,4)$</td><td>$(4,6)$</td><td>$(6,\infty)$</td></tr>
<tr><td>Test #</td><td>-2</td><td>-1</td><td>1</td><td>5</td><td>7</td></tr>
<tr><td>x</td><td>-</td><td>-</td><td>+</td><td>+</td><td>+</td></tr>
<tr><td>$(x-4)$</td><td>-</td><td>-</td><td>-</td><td>+</td><td>+</td></tr>
<tr><td>$(2x+3)$</td><td>-</td><td>+</td><td>+</td><td>+</td><td>+</td></tr>
<tr><td>$(x-6)$</td><td>-</td><td>-</td><td>-</td><td>-</td><td>+</td></tr>
<tr><td>$\frac{x(x-4)(2x+3)}{x-6}$</td><td>+</td><td>-</td><td>+</td><td>-</td><td>+</td></tr>
</table>

</td></tr>
</table>

negative result. See highlighted cells • Remember you are only concerned with the sign not the actual number outcome. We are testing the interval.	
• Use the resulting row (the last row) to answer your question • **Be sure to check which type of brackets are appropriate [,], (, or)**	$\frac{2x^2-5x-12}{x-6} \geq 0$ means which x values result in a positive output. This happens when $\left(-\infty, -\frac{3}{2}\right] \cup [0, 4] \cup \ (6, \infty)$ Note that $x \neq 6$ so it cannot have a square bracket

Your instructor may use a similar method that does not use test points. Both methods work.

Used for the Polarity Table

- Solving inequalities that are not linear
- Finding domain when you are required to solve an inequality that is not linear
- Intervals of increasing and decreasing (Calculus)
- Intervals of concavity (Calculus)

Practice:

Pre-Calculus

1. Find the domain of
   (a) $f(x) = \sqrt{x^2 - x - 6}$

   (b) $g(x) = \frac{2x+3}{\sqrt{6x^2-7x-20}}$

   (c) $h(x) = \frac{x+3}{\sqrt{4-x^2}}$

2. Solve the following inequalities
   (a) $x^2 - 11x + 28 < 0$

   (b) $-2x^3 + 22x^2 - 56x \ \leq 0$

   (c) $\frac{2x^2-8}{-4x+5} \geq 0$

Solutions:

1. Find the domain of

(a) Find the domain of $f(x) = \sqrt{x^2 - x - 6}$

We want $x^2 - x - 6 \geq 0$

Look at $x^2 - x - 6 = 0$ to find intervals.

$$(x-3)(x+2) = 0$$

$$x - 3 = 0 \text{ or } x + 2 = 0$$

$$x = 3 \quad \text{ or } x = -2$$

Use these to make intervals and a table

Interval	$(-\infty, -2)$	$(-2, 3)$	$(3, \infty)$
Test number	-3	0	4
$(x-3)$	-	-	+
$(x+2)$	-	+	+
$(x-3)(x+2)$	+	-	+

We want where $x^2 - x - 6 \geq 0$ so where $(x-3)(x+2) \geq 0$ (positive).
This will happen when $x \in (-\infty, -2] \cup [3, \infty)$.
Notice that the brackets are inclusive brackets because $x = -2 \text{ or } 3$ will be included.

Solution: The domain of $f(x) = \sqrt{x^2 - x - 6}$ is $x \in (-\infty, -2] \cup [3, \infty)$.

(b) Find the domain of $g(x) = \frac{2x+3}{\sqrt{6x^2 - 7x - 20}}$.

We want $6x^2 - 7x - 20 > 0$

Notice that it can no longer equal zero because it is a denominator.

Look at $6x^2 - 7x - 20 = 0$

$$6x^2 - 15x + 8x - 20 = 0$$

$$3x(2x-5) + 4(2x-5) = 0$$

$$(2x-5)(3x+4) = 0$$

$$2x - 5 = 0 \text{ or } 3x + 4 = 0$$

$$x = \frac{5}{2} \quad \text{ or } \quad x = -\frac{4}{3}$$

Use these to make intervals and a table

Interval	$\left(-\infty, -\frac{4}{3}\right)$	$\left(-\frac{4}{3}, \frac{5}{2}\right)$	$\left(\frac{5}{2}, \infty\right)$
Test number	-2	0	3
$(2x-5)$	-	+	+
$(3x+4)$	-	-	+
$(2x-5)(3x+4)$	+	-	+

We want where $6x^2 - 7x - 20 > 0$ (positive) so where $(2x - 5)(3x + 4) > 0$.

Solution: The domain of $g(x) = \frac{2x+3}{\sqrt{6x^2-7x-20}}$ is $x \in \left(-\infty, -\frac{4}{3}\right) \cup \left(\frac{5}{2}, \infty\right)$.

(c) Find the domain of $h(x) = \frac{x+3}{\sqrt{4-x^2}}$

We want $4 - x^2 > 0$ (cannot equal zero as it is a denominator)

To do this find:

$4 - x^2 = 0$

$(2 - x)(2 + x) = 0$

$2 - x = 0 \quad or \; 2 + x = 0$

$2 = x \qquad or \;\; x = -2$

Interval	$(-\infty, -2)$	$(-2, 2)$	$(2, \infty)$
Test number	--3	0	3
$(2 - x)$	+	+	-
$(2 + x)$	-	+	+
$(2 - x)(2 + x))$	-	+	-

So the domain for $h(x) = \frac{x+3}{\sqrt{4-x^2}}$ is $x \in (-2, 2)$.

2. Solve the following inequalities.

(a) Solve $x^2 - 11x + 28 < 0$

$x^2 - 11x + 28 = 0$

$(x - 4)(x - 7) = 0$

$x - 4 = 0 \;\; or \; x - 7 = 0$

$x = 4 \qquad or \; x = 7$

Interval	$(-\infty, 4)$	$(4, 7)$	$(7, \infty)$
Test number	-5	0	8
$(x - 4)$	-	+	+
$(x - 7)$	-	-	+
$(x - 4)(x - 7)$	+	-	+

Solution: $x^2 - 11x + 28 < 0$ is true when $x \in (4, 7)$.

(b) Solve $-2x^3 + 22x^2 - 56x \leq 0$

$-2x^3 + 22x^2 - 56x = 0$

$-2x(x^2 - 11x + 28) = 0$

$-2x(x - 4)(x - 7) = 0$

$-2x = 0 \;\; or \; x - 4 = 0 \;\; or \; x - 7 = 0$

$x = 0 \quad or \;\; x = 4 \quad or \; x = 7$

Interval	$(-\infty, 0)$	$(0,4)$	$(4,7)$	$(7,\infty)$
Test number	-2	1	5	10
$-2x$	+	-	-	-
$(x-4)$	-	-	+	+
$(x-7)$	-	-	-	+
$-2x(x-4)(x-7)$	+	-	+	-

Solution: $2x^3 + 22x^2 - 56x \leq 0$ is true when $x \in [0,4] \cup [7,\infty)$.

(c) Solve $\frac{2x^2-8}{-4x+5} \geq 0$

$\frac{2x^2-8}{-4x+5} = 0$ when $2x^2 - 8 = 0$

$2(x^2-4) = 0$

$2(x-2)(x+2) = 0$

$x - 2 = 0 \; or \; x + 2 = 0$

$x = 2 \quad or \; x = -2$

$\frac{2x^2-8}{-4x+5}$ $Does\ not\ exist$ when $-4x + 5 = 0$

$5 = 4x$

$\frac{5}{4} = x$

Interval	$(-\infty,-2)$	$\left(-2,\frac{5}{4}\right)$	$\left(\frac{5}{4},2\right)$	$(2,\infty)$
Test number	-3	0	$\frac{3}{2}$	3
2	+	+	+	+
$x-2$	-	-	-	+
$x+2$	-	+	+	+
$-4x+5$	+	+	-	-
$\frac{2x^2-8}{-4x+5}$	+	-	+	-

Solution: $\frac{2x^2-8}{-4x+5} \geq 0$ is true when $(-\infty,-2] \cup \left(\frac{5}{4},2\right]$